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Reliability Analysis of a Gracefully Degradable System Using Vague Sets

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ABSTRACT:
Present paper investigates the reliability of a gracefully degradable using vague set. Vague numbers have been explored to find out a crisp number from these vague sets. Precisely, to cover up the involved uncertainties in estimation of failure rates we have obtained a crisp output from a vague input. Vague failure rates have been used to evaluate the reliability of a gracefully degradable system. Vague reliability improves this aspect because of its consideration of two types of membership functions.

KEYWORDS: Vague sets, vague sets, membership function, Non-Membership function, Vague Reliability

1. INTRODUCTION:

Most of research in classical reliability theory is based on binary state assumption for states. Gracefully degradable systems describe a smooth change to a lower state of performance of the system, upon each component failure out of the components existing in parallel redundancy. For such systems, it is therefore unrealistic to assume that the system possesses only two states that is, 'working' or 'failed'. Such systems may be considered working to certain degrees at different states of its performance degradation during its transition from fully working state to completely failed state. The degree may be any real number between 0 (to indicate the system is in failed state) and 1 (to indicate the system is fully operational). Zadeh suggested a paradigm shift from the theory of total denial and affirmation to a theory of grading, to give new concept of sets called Vague sets. Vague sets can express the gradual transition of the system from working state to failed state. The crisp set theory only dichotomizes the system in working state and failed state but Vague set theory can cover up all possible states between a fully working state and completely failed state. This approach to the reliability theory is known as profust reliability, wherein the binary state assumption is replaced by vague state assumption.

Work done in this research paper can be classified into following two folds:

Failure data collection is always a crucial requirement for system Vague reliability assessment. In many situations, where human judgements, evaluation and decisions are important, failure data may not be collected accurately. It might sometimes require linguistic terms (like nearly, approximately, close to, about etc.) for expressing the data value. We also propose a method of failure rate parameter estimation using vague numbers.

The present work discusses a gracefully degradable system having three units. The system is capable of being reconfigured, upon the failure of unit and therefore starts functioning in a degraded state.

In fact failure rate is one of the important parameter in reliability theory and its estimation involves uncertainties of different kinds. In some cases the relationship between the failure mechanism and the failure rate function may be used in making a choice of failure distribution. Sometimes two or more types of failures occur at once. It is the exponential distribution that has been mostly explored by the researchers for failure distribution because it has a number of desirable mathematical properties. It has constant failure rate. To estimate the constant parameter λ of the exponential distribution, the system is to be tested for failure times under certain desired operating conditions. It may impart different failure characteristics in different trials.

Assignment of a crisp number to the parameter λ at this stage involves uncertainties in actual observations of failure time and also in making statistical approximation for getting a single value from several close options. Vague numbers have been explored to find out a crisp number from these vague sets. Precisely, to cover up the involved uncertainties in estimation of failure rates we have obtained a crisp output from a vague input.

2. CONCEPTS OF VAGUE SET:

(i) Vague set. A vague set $\tilde{A}$ in the universe of discourse X is characterized by two membership functions as:

(1) truth membership function

$\mu_{\tilde{A}} : X \to [0,1]$ and

(2) false membership function

$\nu_{\tilde{A}} : X \to [0,1]$.

The grade of membership for any element *x* in the vague set is bounded by a sub interval [$\mu_{\tilde{A}}(x)$,1-$\nu_{\tilde{A}}(x)$] of [0,1] where the grade $\mu_{\tilde{A}}(x)$ is called the lower bound of membership grade of *x* derived from favourable evidence for *x* and $\nu_{\tilde{A}}(x)$ is the lower bound of membership grade on the negation of *x* derived from the evidence against *x*, where $\mu_{\tilde{A}}(x)+\nu_{\tilde{A}}(x)\leq 1$. The interval, [$\mu_{\tilde{A}}(x)$,1-$\nu_{\tilde{A}}(x)$] is called the vague value of x in $\tilde{A}$. In the extreme case of equality where $\mu_{\tilde{A}}(x)=$ 1- $\nu_{\tilde{A}}(x)$, the vague set reduces to the Vague set with interval value of the membership grade reducing to a single value $\mu_{\tilde{A}}(x)$. In general, however,

$\mu_{\tilde{A}}(x) \leq$ Exact membership grade of $x \leq 1$ - $\nu_{\tilde{A}}(x)$.

Expressions given below can be used to represent a vague set $\tilde{A}$ for finite, countable and uncountable universe of discourse X respectively:

$$\tilde{A}=\sum_{k=1}^{n}[\mu_{\tilde{A}}(x_k),1-\nu_{\tilde{A}}(x_k)]/x_k \, , \tilde{A}=\sum_{k=1}^{\infty}[\mu_{\tilde{A}}(x_k),1-\nu_{\tilde{A}}(x_k)]/x_k$$

$$\tilde{A}=\int_{x\in X}[\mu_{\tilde{A}}(x),1-\nu_{\tilde{A}}(x)]/x \, .$$

(II) BASIC CONCEPT OF VAGUE NUMBER:

Now let us assume that the trapezoidal membership function is given by the following expression:

$$\mu_A(x)=\begin{cases} 0 & , \ for \quad x\leq a \ \ or \ \ x\geq d \\ \dfrac{x-a}{b-a} & , \ for \quad a\leq x\leq b \\ 1 & , \ for \quad b\leq x\leq c \\ \dfrac{d-x}{d-c} & , \ for \quad c\leq x\leq d \end{cases}$$

$$= (a, b, c, d)$$

Similarly, we can also generate a non-membership function

$$\nu_A(x)=\begin{cases} \dfrac{(b-x)}{b-a} & , \ for \quad a\leq x\leq b \\ 0 & , \ for \quad b\leq x\leq c \\ \dfrac{(x-c)}{d-c} & , \ for \quad c\leq x\leq d \\ 1 & , \ for \quad otherwise \end{cases}$$

A trapezoidal vague number $\tilde{A}$ denoted by [(a, b, c, d); $\mu_A(x), 1-\upsilon_A(x)$] is characterized by a pair of membership functions:

3. BASIC CONCEPT OF RELIABILITY:

It is the probability that the system will be able to operate successfully without failure during a given time interval (0, t] under the stated operating conditions symbolically, it is defined as

$$R(t) = P[X(u) = 1 \; \forall \; 0 < u \leq t / X(0) = 1]$$

Mathematically, if T denotes the life time of the system, then the reliability or survival function of the device at time t is

$$R(t) = P[T > t] = \int_t^{\infty} f(u)\, du$$
$$= \begin{cases} 1 - F(t), & t > 0 \\ 1 \quad , & t = 0 \end{cases}$$

where F(.) is the c.d.f. of T.
Thus reliability is a function of time t. It should be also noted that
(i) $R(0) = 1$, since the device is assumed to be operable at time t =0
(ii) $R(\infty) = 0$, since no device can work forever without failure
(iii) $R(t)$, is a non-increasing function of time between limits 0 to 1

4. FAILURE RATE ESTIMATION:

Failure / Repair rates are important parameters in the estimation of reliability characteristics of any system. A small error in failure rate may lead to over / under estimation of system reliability. For systems having very sensitive applications, this risk must be avoided to maximum possible extent. A standard method for determining a failure rate parameter is maximum likelihood utilizing estimation from multiple data sets.
We propose a method to estimate failure rate parameter by using the concepts of trapezoidal vague numbers. The philosophy of the method is based on following steps:
(a) Failure rate is first estimated according to existing procedure. The process must be repeated so that more than one number is available for estimating the failure rate.
(b) Numbers obtained in (a) are replaced by vague numbers close to each.
(c) Get a single vague number by taking the union of the vague numbers obtained through step (b).
(d) Defuzzify the membership and non-membership for vague number of step (c), to get a single crisp (non-vague) number as the final estimate for failure rate.
To demonstrate the process of vague failure rate estimation, it would be in the fitness of the things, if we use same crisp failure rates for vague failure rate estimation and also for vague reliability estimation in this paper. This will be beneficial in making a comparative study of the reliability estimates in the two cases.
Let λ_1=0.0003, λ_2=0.0006 and λ_3=0.0018 be three numbers obtained as the estimated failure rates of the components of unit, by the existing method in three repetitions of the process. Instead of taking their mean λ= 0.001 as the final value for failure rate, step (b) suggests to define three Vague numbers $\tilde{\lambda}_1$ $\tilde{\lambda}_2$ and $\tilde{\lambda}_3$ about 0.0003, 0.0006 and 0.0018 respectively.

$$\mu(\tilde{\lambda}') = \begin{cases} \dfrac{\lambda}{.0001} & 0 \leq \lambda \leq .0001 \\ 1 & .0001 \leq \lambda \leq .0004 \\ \dfrac{.0006-\lambda}{.0003} & .0003 \leq \lambda \leq .0006 \end{cases} \qquad \upsilon(\tilde{\lambda}'_1) = \begin{cases} \dfrac{.0004-\lambda}{.0003} & 0.0001 \leq \lambda \leq .0004 \\ \dfrac{\lambda-.0006}{.0003} & .0003 \leq \lambda \leq .0006 \\ 0 & 0 \leq \lambda \leq .0001 \end{cases}$$

$$\mu(\tilde{\lambda}'') = \begin{cases} \frac{\lambda - .0004}{.0003} & .0004 \le \lambda \le .0007 \\ 1 & .0007 \le \lambda \le .0010 \\ \frac{.0012 - \lambda}{.0002} & .0010 \le \lambda \le .0012 \end{cases} \qquad \upsilon(\tilde{\lambda}'') = \begin{cases} \frac{.0010 - \lambda}{.0003} & .0007 \le \lambda \le .0010 \\ \frac{\lambda - .0012}{.0002} & .0010 \le \lambda \le .0012 \\ 0 & 0.0004 \le \lambda \le .0007 \end{cases}$$

$$\mu(\tilde{\lambda}''') = \begin{cases} \frac{\lambda - .0012}{.0004} & .0012 \le \lambda \le .0016 \\ 1 & .0016 \le \lambda \le .0018 \\ \frac{.0024 - \lambda}{.0006} & .0018 \le \lambda \le .0024 \end{cases} \qquad \upsilon(\tilde{\lambda}''') = \begin{cases} \frac{.0018 - \lambda}{.0002} & .0016 \le \lambda \le .0018 \\ \frac{\lambda - .0024}{.0006} & .0018 \le \lambda \le .0024 \\ 0 & .0012 \le \lambda \le .0016 \end{cases}$$

As suggested in step (c), the Vague union of $\tilde{\lambda}_1(x)$, $\tilde{\lambda}_2(x)$ and $\tilde{\lambda}_3(x)$ is given by $\tilde{\lambda}$ where,

$$\tilde{\lambda}(x) = \begin{cases} \frac{x}{.0004} & \text{if } 0 \le x < .0004 \\ 1 & \text{if } .0004 \le x < .0006 \\ \frac{.0009 - x}{.0003} & \text{if } .0006 \le x < .000729 \\ \frac{x - .0005}{.0004} & \text{if } .000729 \le x < .0009 \\ 1 & \text{if } .0009 \le x < .0011 \\ \frac{.0016 - x}{.0005} & \text{if } .0011 \le x < .001267 \\ \frac{x - .001}{.0004} & \text{if } .001267 \le x < .0014 \\ 1 & \text{if } .0014 \le x < .0016 \\ \frac{.0019 - x}{.0003} & \text{if } .0016 \le x < .0019 \end{cases}$$

Using the centroid method for defuzzification of λ, we consider the area covered by the Vague number and return the center of the gravity of the covered area as the required non-Vague number.

$$\lambda \text{ (Non-Vague number)} = \frac{\int_C x\,\tilde{\lambda}(x)\,dx}{\int_C \tilde{\lambda}(x)\,dx}$$

Using the defuzzification of $\tilde{\lambda}(x)$ yields the crisp value .00112 for $\tilde{\lambda}$, from which we can get the Vague reliability evaluation of the system.

5. VAGUE RELIABILITY ESTIMATION:

Let A (n) be a unit having n parallel components. The system is fully working if all the n components of the system A are working. Failure of each component of unit A causes the degradation of the system and the system continues to work with increased degradation until it reaches to a desirable threshold.

Let k be the minimum number of working components of unit A, which is required to have the system in "working" state, Then according to conventional reliability theory, the reliability of such a system may be given as,

$$R_k(t) = \sum_{k=K}^{n} \binom{n}{k} [P(t)]^k [1 - P(t)]^{n-k}$$

where P (t) is the probability that a component is working at time t. Now if λ is constant failure rate of the component of unit A, then the above expression can be given as,

$$R_k(t) = \sum_{k=K}^{n} \binom{n}{k} [e^{-\lambda t}]^k [1 - e^{-\lambda t}]^{n-k}$$

Since, the degradable systems may be considered as working and failed to certain degrees between "fully working" and "fully failed" state, therefore, the conventional reliability theory fails to describe the reliability characteristics of such type of systems. So, we have to look at Vague set theory to overcome such problems.

Let the universe of discourse be U= {S_0, S_1, S_2 ....S_n}. On this universe we can define a Vague success state S and a Vague failure state F as below:

$$S = \{S_i, \mu_S(S_i); i = 1,2,3..........n\} \quad S = \{S_i, \upsilon_S(S_i);\ i = 1,2,3................n\}$$

$$F = \{S_i, \mu_F(S_i); i = 1,2,3..........n\} \quad F = \{S_i, \upsilon_F(S_i);\ i = 1,2,3................n\}$$

Let $U_\tau = \{m_{ij}, i, j = 1,2,3..........., n\}$ be another universe of discourse, where m_{ij} is the transition from state S_i to state S_j . Let the membership grade of Vague transition m_{ij} be

$$\mu_{T_{SF}}(m_{ij}) = \begin{cases} \beta_{F/S}(S_j) - \beta_{F/S}(S_i) \ if\ \beta_{F/S}(S_j) > \beta_{F/S}(S_i) \\ 0 \qquad\qquad otherwise \end{cases}$$

and

where $$\beta_{F/S}(S_i) = \frac{\mu_F(S_i)}{\mu_F(S_i) + \mu_S(S_i)};\ i = 1,2,..........n$$

Then the Vague set of transition can be defined by,

$$T_{SF} = \left\{m_{ij}, \mu_{T_{SF}}(m_{ij}); i, j = 1,2,.............n\right\}$$

In this way T_{SF} represents the transition from Vague success state to Vague failure state and can be viewed as a Vague event. The profust reliability is defined as,

$$R(t_0, t_0 + t) = P\left\{T_{SF}\ does't\ occur\ during\ the\ time\left[t_0, t_0 + T\right]\right\}$$

$$= 1 - \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_{T_{SF}}(m_{ij}) P\{m_{ij}\ occurs\ during\left[t_0, t_0 + T\right]$$

where, m_{ij} is the Vague transition from state S_i to state S_j without passing through any intermediate states.

In probist reliability theory, at any time t the values for membership functions of system success S and system failure F are either zero or one, and satisfy

$$\mu_S(S_i) = 1 - \mu_F(S_i);\ i = 1,2,............., n.$$

Since the system passes through all intermediate states, therefore, S_j cannot move to any state other than S_{j-1}. Thus we have,

$$R(t_0, t_0 + t) = 1 - \sum_{j=1}^{n} \mu_{T_{SF}}(m_{(j+1)j}) P\{m_{(j+1)j}.occuring\ during\left[t_0, t_0 + t\right]\}$$

Let S_n and S_j denote the states of the system at time t_0 and $t_0 + t$ respectively then it is obvious that the transition $m_{(n-1)n}, ...m_{(j+1)j}$ have occurred during [$t_0, t_0 + t$]. So,

$$\mu_{T_{SF}}(m_{ij}) = \mu_{T_{SF}}(m_{ik}) + \mu_{T_{SF}}(m_{jk}) \quad if \quad \mu_F(S_i) < \mu_F(S_k) < \mu_F(S_j)$$

Further, we note-

$$\mu_{T_{SF}}(m_{ij}) = \mu_{T_{SF}}(m_{ik}) + \mu_{T_{SF}}(m_{jk}) \quad if \quad \mu_F(S_i) < \mu_F(S_k) < \mu_F(S_j)$$

Then we have,

$$R(t_0, t_0 + t) = 1 - \sum_{j=1}^{n-1} \mu_{T_{SF}}(m_{ij}) P\{at\, time\, t_0 + t\, the\ system\ is\ in\ S_j\}$$

$$= \sum_{j=1}^{n-1} \mu_{T'_{SF}}(m_{ij}) P\{at\, time\, t_0 + t\, the\ system\ is\ in\ S_j\}$$

Where, $\mu_{T'_{SF}}(m_{ij}) = 1 - \mu_{T_{SF}}(m_{ij})\ ; i, j = 1, 2, ...n$

For the present system we define one Vague success state S and one Vague failure state F. Let us have linear degradation of unit so that membership functions for success and failure states can be defined as below:

$$\mu_S(S_i) = \frac{i}{N} \quad ; i = 0, 1, 2..........N$$

$$= \frac{i}{3} \quad ; i = 0, 1, 2, 3.$$

$$and \quad \mu_F(S_i) = \frac{N-i}{N} \quad ; \ i = 0, 1, 2...........N$$

$$= \frac{3-i}{3} \quad ; \ i = 0, 1, 2, 3.$$

Then, we have,

$$\mu_{T_{SF}}(m_{ij}) = \begin{cases} \dfrac{i-j}{N} & , \ if \ i > j \\ 0 & , \ if \ i \le j \end{cases}$$

$$= \begin{Bmatrix} {i-j}/{3} & if & i > j \\ 0 & if & i \le j \end{Bmatrix}$$

Also, we have $P_N(0) = 1$

So, the system profust reliability becomes,

$$R(t) = \sum_{j=1}^{N} \frac{j}{N} P_j(t)$$

$$= \sum_{j=1}^{N} \frac{j}{N} \ c^{N-j} \ e^{-\lambda_j t} \ \frac{!N}{!j \ !N-j} \left(1 - e^{-\lambda t}\right)^{N-j}$$

6. DISCUSSION AND INTERPRETATION:

The classical set theoretic approach does not seem very effective to cover up most of the uncertainties occurred in the data and various statistical methods used for failure rate estimation and reliability evaluation of a system. Here, in this Paper we have introduced a new approach for failure rate estimation based on vague set theory, which is more realistic approach in case of system behaviour. Also the degradation of the system is considered to be Vague in nature rather than crisp i.e. various system states have been taken between a fully working and fully failed state to evaluate the reliability of the system. The classical reliability of the system, which is a particular case of vague reliability, has also been evaluated.

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